DETECTION OF BOILING BY NOISE ANALYSIS

By

P. KRISHNAMOORTHY

DEPARTMENT OF NUCLEAR ENGINEERING AND
TECHNOLOGY PROGRAMME
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JANUARY, 1986

DETECTION OF BOILING BY NOISE ANALYSIS

A Thesis Submitted
in Partial Fulfilment of the Requirements
for the Degree of

MASTER OF TECHNOLOGY

By
P. KRISHNAMOORTHY

to the

DEPARTMENT OF NUCLEAR ENGINEERING AND
TECHNOLOGY PROGRAMME
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

JANUARY, 1986

TO MY BELOVED PARENTS

CERTIFICATE

This is to certify that the work "DETECTION OF BOILING BY NOISE ANALYSIS", has been carried out by Mr. P. KRISHNA MOORTHY under our supervision and that it has not been submitted elsewhere for a degree.

(P. Munshi)
Lecturer
Dept. of Nuclear Engg.
I.I.T. Kanpur
India

(K. Sriram)
Professor and Head
Department of Nuclear Engg.
Indian Institute of Technology
Kanpur, India

December, 1985

ACKNOWLEDGEMENTS

It is a pleasure to thank Dr. K. Sri Ram and Mr. Prabhat Munshi for their diligent guidance during the course of the present study. I should also thank Mr. S.S. Pathak and Mr. R.S. Tripathi for their helpful cooperation in making this experiment as a success.

I am indebted to the other faculty members whose encouragement was invaluable. I should also thank my friends circle in Hall No.4 whose association gave me inspiration and pleasure during my stay here.

I must thank finally Mr. U.S. Mishra for typing my thesis patiently and neatly.

-P. KRISHNAMOORTHY

TABLE OF CONTENTS

		Page
	LIST OF TABLES	vi)
	LIST OF FIGURES	vii)
	NOMENCLATURE	viii)
	ABSTRACT	x)
CHAPTER I	INTRODUCTION	1
CHAPTER II	EXPERIENCES WITH NOISE ANALYSIS	3
2.1	Experience in Loose Parts Monitoring and Operating Nuclear Power Plants	3
2.2	Boiling Detection in Fast Reactors. Studies Performed in France	4
2.3	Literature Survey	5
CHAPTER III	SIGNAL PROCESSING	9
CHAPTER IV	METHOD OF MEASUREMENT	10
CHAPTER V	MATHEMATICAL ANALYSIS OF NOISE	12
5.1	Variance	12
5.2	Correlation Functions and Power Spectral Density Functions	14
5.3	Cross Correlation Function and its Properties	19
CHAPTER VI	ELECTRONIC INSTRUMENTS DESCRIPTION	21
6.1	Piezo-electric Transducer	21
6.2	Charge Amplifier (Model 2735)	21
6.3	Tunable Band-Pass Filter	22
6.4	Correlation and Probability Analyser	22
6.4.1	Correlation	23
6.5	Fourier Transform Analyser (Model SAI-470)	23
6.5.1	General Functional Description	23
CHAPTER VII	EXPERIMENTAL EQUIPMENT AND PROCEDURE	27
CHAPTER VIII	EXPERIMENTAL OBSERVATIONS AND TABULATIONS	31

			Page
CHAPTER	IX	RESULTS AND DISCUSSIONS	37
CHAPTER	X	CONCLUSIONS AND RECOMMENDATIONS	41
	10.1	Conclusions	41
	10.2	Recommendations	41
		REFERENCES	42
		APPENDIX-I	44
		APPENDIX-II	47

LIST OF TABLES

Table	Description	Page
1	Frequency Spectrum of Air Injection into Stationary Water	31
2	Frequency Spectrum for 3218 cc/sec Water Flow Rate	32
3	Frequency Spectrum for 352.61 cc/sec Water Flow Rate	33
4	Frequency Spectrum for 360.86 cc/sec Water Flow Rate	34
5	Frequency Spectrum for 407.15 cc/sec Water Flow Rate	35
6	Frequency Spectrum for 431.85 cc/sec Water Flow Rate	36
7	Frequency Shift with Water Flow Rates	38

LIST OF FIGURES

Figure	Description	Page
1	Schematic Diagram of Electronic Instruments Used	29
2	Arrangement of Chamber with Transducer	30
3	Plot of Frequency Versus $(\text{Velocity})^4$ of Liquid	40
4.(a)	Bubble Formation	45
4.(b)	Forces on a Free Bubble within a Liquid	45

NOMENCLATURE

x	Random variable
$\bar{x}$	Average value of x
t	Time
$x(t)$	Time varying function
$y(t)$	Time varying function
$\bar{x}(t)$	Average of time varying function
N	Number of data points. Also number of trials in a statistical process
T	Duration of data
τ	Time lag in a correlation function; also counting time
σ	Standard deviation
σ^2	Variance
$\phi(\tau)$	Correlation function
$\phi'(\tau)$	Normalized correlated function
i	$\sqrt{-1}$
$\Phi(f)$	Power spectral density
Δt	Spacing of digitized data points
X	Fourier amplitude of x(t)
$p(x)$	Probability density function
w	$2\pi f$; frequency in Radians
f	Frequency in c.p.s.
$M=\tau_m/\Delta t$	Maximum number of data points lagged in computing a correlation

$\tau_m = M\Delta t$	Maximum time for correlation considered, i.e. total time delay span
S	Surface tension of liquid dynes/cm
Θ	Angle of contact between water and glass in degrees
P	Pressure - dynes/cm^2
r	Radius of bubbles in cms
R	Radius of hole in the chamber for air injection in mm
v	Velocity of liquid in cm/sec
ρ	Density of liquid in gms/cm^3
μ	Viscosity of liquid

Subscripts

xx	Auto correlation
xy	Cross correlation of x and y
k	Index of summation
f	Liquid
g	Gas (air)

ABSTRACT

The present work is based on simulation of boiling by air injection into water medium to obtain the frequency response of bubbles for air flow rate range 128.9 cc/sec to 644.5 cc/sec and water flow rate range 321.88 cc/sec to 431.85 cc/sec.

During the bubble collapse the acoustic signals are picked up by a piezo electric transducer. The signals are amplified, filtered and then it is auto-correlated. To get frequency response, the auto-correlation is fed into the Fast Fourier Transform Analyser. The power spectra obtained for the system contain information about the bubble-size, shifting frequency as a function of velocity of the liquid.

The dependence of predominant bubble frequency is found to be directly proportional to the 4-th power of flow velocity.

CHAPTER I

INTRODUCTION

The term noise when applied to reactor study, it means random phenomena like the fluctuations in neutron population. Analysis of noise sources in a power reactor can give vital information about the state of the reactor without disturbing the normal operation. According to Fourier's theorem, a function of time is capable of being characterized by the superposition of sine waves of various frequencies having appropriate amplitudes and phases. The amplitudes that are present at various frequencies therefore become an alternative method of characterising the noise.

In general a random variable x, which is a function of time can be represented as

$$x(t) = \int_{-\infty}^{\infty} X(f)\ \exp(ift)df$$

where $X(f)df$ is the complex amplitude of the oscillation in the region between f and f+df. The behaviour of magnitude, $|X(f)|^2$, with frequency is one of the most useful methods of characterising the noise.

Some of the applications of noise analysis in reactors are power monitoring, coolant flow-rate and void fraction measurements, detection of nucleate boiling and structural vibrations. The same noise analysis techniques are

used in industries. Where a large capital investment is involved in a plant, it is expensive to stop production in order to do testing. Hence noise analysis is a powerful tool for monitoring a system during its on-line process.

The objectives of the present work are:

(1) To demonstrate that noise techniques can be used to obtain information about nucleate boiling.

(2) To show that noise techniques can be used to determine the frequency response (spectral density) of bubbles in nucleate boiling for various flowrates of coolant.

The present work is based on simulation of boiling by air injection into water medium and to obtain the frequency response of bubbles. The power spectra obtained for the system contain information about the bubble-size, shifting frequency as a function of velocity of the coolant.

One of the principal advantages of the correlation techniques which is used in this experiment is to extract signals buried in noise.

The experimental equipment and procedure are described in Chapter (7).

CHAPTER II

EXPERIENCES WITH NOISE ANALYSIS

2.1 Experience in Loose Parts Monitoring and Operating Nuclear Power Plants

Some of the earlier investigations are given below:

The monitoring of the french "PHENIX" LMFBR [1] dynamic behaviour is assumed by periodical recordings of signal fluctuations of neutronic noises and vibration measurements. The frequency analysis of these recordings made by the power spectral density and the coherence function analysis allowed the interpretation and monitoring of most noise sources caused by pump functioning and structure movements under sodium flow. The results that are obtained are as follows:

(1) A movement of the concrete slab of the primary circuit at the fundamental frequency corresponding to the pumps rotary speed.

(2) A low level vibration on a control rod drive corresponding to an incipient defect.

(3) Emergence of vibrations on the core cover during a working cycle.

(4) A resonance corresponding to the first vibrational mode of the fuel assemblies under the cooling sodium flow.

The evidence of the vibratory movement has been obtained by the systematic calculation of the coherence function between the available neutronic noise measurements and the vibratory transducers. The experiences gained on PHENIX vibration measurements are included in the SUPERPHENIX monitoring system.

Fry and Robinson [2] observed a sudden peak in their library of neutronic noise spectra. The peak was attributed to the control rod bearing failure. The peak disappeared when the defect was removed. Hence the above example illustrates how noise spectra were used to find the mechanical failure of a system.

2.2 Boiling Detection in Fast Reactors by Noise Analysis. Studies Performed in France

The onset of accidental conditions particularly the boiling of sodium is generally related to background noise modifications in an operating reactor. The detection of in-core boiling may be achieved by three different analysis methods. They are: (1) neutron noise, (2) acoustic noise and (3) temperature fluctuations at the subassembly outlets. On these three methods experiments were conducted in RAPSODIE reactor in France. These results are as follows [3].

Thermohydraulic analysis of boiling in CFNa bundle have demonstrated that two main regimes may appear:

- hot spot boiling
- single bubble regime.

The transition between these two regimes is fairly sharp. The behaviour of the boiling process is rather insensitive to thermohydraulic parameters (flow, heat flux, geometry).

It appears that

- Thermal noise detection is especially effective in hot spot boiling conditions.
- Acoustic noise is directly sensitive to hot spot boiling and also, modulated by the single bubble regime.
- Void effect measures either by pressure or flow fluctuations or neutron power fluctuations, is only detected in the presence of single bubble.

The above mentioned three methods are complementary and correlation between them leads to a reliable boiling detection system.

2.3 Literature Survey

Strasberg [4] has given that gas bubbles when entrained in water or other liquid can generate high sound pressures in the liquid. Significant sound pressures are associated only with volume pulsations of the bubble whereas oscillations in the shape of the bubble do not result in appreciable sound. He has done calculations of sound

pressures resulting from excitation of volume pulsations by the following mechanisms:

(i) Bubble coalescence, or division;

(ii) The motion of a free stream of liquid containing entrained bubbles past an obstacle; and by the flow of liquid with entrained bubbles through a pipe past a constriction. The calculation of the sound pressure generated by bubble formation has been verified by measurements with bubbles formed at a nozzle.

Claytor [5] has injected steam into sodium to measure their acoustic characteristics. Steam was injected through small swaged leaks in Type 304 stainless steel tubing. The swaged portion of the leak was typically 2.5 cm long. During a leak, acoustic noise was measured simultaneously with lithium niobate microphone and accelerometers. He repeated with inert gas injection into sodium. And concluded that frequency peaks upto 20 KHz are due to gas bubble resonance. At frequencies between 20 and 100 KHz, combustion noise is probably the main source of noise for steam injections. Above 120 KHz the mechanism of sound generation by steam and inert gas injections are not fully understood. It is concluded that noise generation is dependent of sodium cover gas pressure.

Claytor [6] also conducted an experiment at ANL for various sodium flow at various steam injection rates. He

concluded that background noise at 2 KHz peak is due to argon bubbles at the orifice of the tube which was to keep the injector from plugging. An increase of 8.5 dB over the background level is observed at 2 KHz when steam is injected. The noise produced at 2 KHz was possibly attribute to acoustic monopole radiation from hydrogen bubbles oscillating in the volume mode after being formed by the steam-sodium reactions. Finally, it was shown that bubble-generated sound due to peaks are prevalent at low frequencies ($f < 10$ KHz) as is the ambient noise which is present in a LMFBR-SG due to flow.

Ying [7] developed an acoustic detection system to prevent serious damage to the tubes that would result from sodium water reactions. The leak can be identified from the acoustic spectrum of noise generated during sodium-water reactions which produce hydrogen gas with sodium hydroxide or sodium monoxide. The significant audible sound is the sound radiated from the damped oscillations of hydrogen bubbles in liquid and the frying noise generated at the interface of sodium and water have peaks in the acoustic spectra at frequencies below 1.5 KHz and near 2 KHz respectively due to exothermic heat of the chemical reactions.

DE [8] measured the distribution of time intervals between incipient bubbles in a venturi and concluded from the observations that cavitation noise observed is neither the result of an impulsive random pulse nor a purely cyclic

process. Such noise results from bubble clusters incepted periodically in a train (3 to 5 appearances) and trains appearing at random. This is possibly due to the back action of bubble nucleation on flow turbulence. The above observations provide further understanding of the fundamental process and preliminary data that indicate possibilities for detection schemes in monitoring systems for incipient two phase flow.

Gavrilov [9] showed the results of measurements of the sound attenuation induced by gas bubbles can be used to find the size distribution of the bubbles in a long-standing water and tap water.

CHAPTER III

SIGNAL PROCESSING

The most simple descriptor used for getting information from fluctuating data is the 'mean' value that expresses the steady flow of information in the data or signal.

To analyse the signals coming from more complex systems and the limitations of the simple mean and/or variance parameters led to the evolvement of better statistical descriptors such as correlation functions, power spectral density, probability density distribution etc. For these calculations we require analog and digital correlators, spectrum analysers based on Fast Fourier Transform Techniques.

The use of fast Fourier Transform Techniques developed in mid 1960's has been really most revolutionising and many of the tasks that were possible only off-line before their advent, can be performed on-line today in the plants. Development of digital filters, fast analog to digital convertor and the use of multiplexer have been useful in studying the several signals simultaneously along with their cross correlation.

The availability of analog tape recorders have made storage of data easy and hence the scope of extensive research on available signals.

CHAPTER IV

METHODS OF MEASUREMENT

There are two methods of noise recording and analysis:

(1) Analog (or continous), and

(2) Digital (or discrete).

Continuous recording may be accomplished by standard chart recorders or by magnetic tape recording. Having once recorded an experiment on magnetic tape, it can be return and reanalysed at any time and as often as is desired. Data so recorded is easily stored and is compatible with both analog and digital methods of analysis. Also by using different recording and playback speeds, one can achieve various time transformations in the analysis. Although continuous signals can be recorded such that the degree of magnetization is proportional to the signal (direct amplitude recording), it is usually preferable to record the frequency modulation of a carrier at constant amplitude. Auto-correlation functions can be computed from the playback-head signals from identical channels by varying the length of tape between the two heads to achieve various delays.

It is very common in reactor installations to have the reactor power and other plant variables recorded on X-Y plotters. The charts give the basis for off-line digitizing i.e. after reactor operation. It provides a permanent record of

the experiment. Experimentalists may wish to 'see' what is happening during the experiment.

When digital analysis of a continuous signal is desired, an analog-to-digital conversion must be performed.

The schematic diagram of signal processing is given in Chapter (7). This is an on-line electronic analysis i.e. during plant operation.

CHAPTER V

MATHEMATICAL ANALYSIS OF NOISE

5.1 Theory

Variance

The brief treatment of random signals (noise analysis) which is given in this section follows essentially that of Thie [10] and Rice [11]. For a random process, the n-th moment having probability density function p(x), is given by

$$\overline{x^n} = \frac{\int_{-\infty}^{\infty} x^n \, p(x) \, dx}{\int_{-\infty}^{\infty} p(x) \, dx} \qquad (1)$$

or, for a discrete distribution,

$$\overline{x^n} = \frac{\sum_k x_k^n \, p(x_k)}{\sum_k p(x_k)} \qquad (2)$$

Equation (2) suggests that $\overline{x^n}$ may be determined experimentally from a sequence of a large number, N, of data values, x_k, since these will tend to distribute themselves according to $p(x_k)$:

$$\overline{x^n} = \frac{1}{N} \sum_{k=1}^{N} x_k^n ,$$

or, for continuous data,

$$\overline{x^n} = \frac{1}{T} \int_{-T/2}^{T/2} x^n \, dt.$$

where T is the duration of the experiment for which $x(t)$ is available. It is evident that the moments are average values of various powers of the random variable.

The first moment, $\bar{x}$, is the average value. For most noise applications it is convenient to choose the co-ordinates such that $\bar{x}$ will be zero. But to avoid loss of generality, it need not be zero here. That being the case, it is more convenient to evaluate moments with respect to $\bar{x}$,

$$\overline{(x-\bar{x})^n} = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^n \, p(x)dx}{\int_{-\infty}^{\infty} p(x) \, dx}$$

For n = 2, the result is called the "variance". The square root of the variance is termed the standard deviation σ :

$$\sigma^2 = \frac{\int_{-\infty}^{\infty} (x-\bar{x})^2 \, p(x)dx}{\int_{-\infty}^{\infty} p(x) \, dx} = \frac{1}{T} \int_{-T/2}^{T/2} (x-\bar{x})^2 \, dt$$

For digital data,

$$\sigma^2 = \frac{\sum_{k=1}^{N} (x_k-\bar{x})^2 \, p(x_k)}{\sum_{k=1}^{N} p(x_k)} = \frac{1}{N} \sum_{k=1}^{N} (x_k - \bar{x})^2 .$$

Now,

$$\frac{1}{N}\sum_{k=1}^{N}(x_k - \bar{x})^2 = \frac{1}{N}\sum_{k=1}^{N}(x_k^2 - 2\bar{x}\,x_k + \bar{x}^2)$$

$$= \sum_{k=1}^{N}\frac{x_k^2}{N} - 2\bar{x}\sum_{k=1}^{N}\frac{x_k}{N} + \sum_{k=1}^{N}\frac{\bar{x}^2}{N}$$

$$= \overline{x_k^2} - 2\bar{x}^2 + \bar{x}^2$$

$$\sigma^2 = \overline{x_k^2} - \bar{x}^2 .$$

Therefore standard deviation,

$$\sigma = [\overline{x_k^2} - \bar{x}^2]^{1/2}$$

The standard deviation is also referred to as the "rms" (root mean square) value. The rms value is very commonly used as a quantitative measure of the amount of noise, since it is easy to compute, it has particular significance in those distributions that are gaussian, and it is easy to measure experimentally.

5.2 Correlation Functions and Power Spectral Density Functions

We consider a physical process which gives rise to a time-varying signal x(t). The signal may be simple or complex periodic or have the character of noise, that is random-varying. If this signal is delayed, thereby obtaining

the signal $x(t-\tau)$, which is identical to $x(t)$ but just delayed in time, and then the product of $x(t).x(t-\tau)$ is averaged over a sufficiently long time, the auto-correlation function, $\phi_{xx}(\tau)$, for the signal $x(t)$ will be determined. Mathematically, the auto-correlation function is defined as,

$$\phi_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\ x(t-\tau)\ dt \tag{1}$$

Note also that $\phi_{xx}(\tau)$ is an even function, that is $\phi_{xx}(\tau) = \phi_{xx}(-\tau)$, and therefore eqn.(1) may be written as

$$\phi_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\ x(t+\tau) dt \tag{2}$$

For digital signals having N discrete data points, eqn.(2) can be re-written as replacing integral by summation

$$\phi_{xx}(\tau) = \lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^{N} x(t_k)\ x(t_k+\tau) \tag{3}$$

The following are the properties of this function, $\phi(\tau)$:

1. $\phi_{xx}(\tau) = \phi_{xx}(-\tau)$:

This states that the Auto-correlation function is an even function and is symmetrical about $\tau = 0$.

2. $\phi_{xx}(0)$ = mean square value

This states that at $\tau = 0$ the total signal power (AC and DC) is represented.

3. $\phi_{xx}(\infty)$ = (average value)^2

This states that the value for large values of τ is approaching the DC power of the signal.

4. $\phi_{xx}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(f)\, e^{i2\pi f}\, df$.

This states that the Auto-correlation function and the power spectral density $\Phi(f)$ form Fourier Transform pairs.

5. In Auto-correlation calculations, phase information is lost.

The normalized auto-correlation function is defined w.r.t. to deviation from the mean and is given by

$$\phi'_{xx}(\tau) = \lim_{T \to \infty} \frac{1}{2\,\sigma_{xx}^2 T} \int_{-T}^{T} [x(t)-\bar{x}][x(t+\tau) - \bar{x}]\,dt \qquad (4)$$

For digital techniques,

$$= \lim_{N \to \infty} \frac{1}{\sigma_{xx}^2 N} \sum_{k=1}^{N} [x(t_k)-\bar{x}]\cdot[x(t_k+\tau) - \bar{x}]$$

where $\bar{x}$ is the mean value of the time varying signal over one period of a periodic signal or over the discrete number of

digitized values of a random signal, and σ_{xx}^2 is the variance of the signal $x(t)$. In practice, with N data points spaced Δt apart, $\phi_{xx}(\tau)$ is calculated for τ upto $\tau_m = M\Delta t \ll N\,\Delta t$ where M is the maximum number of correlation points.

τ_m — is the total time delay span which is equal to M t.

$$\phi'_{xx}(\tau) = \frac{\frac{1}{N-\tau/\Delta t} \sum_{k=1}^{N-\tau/\Delta t} (x_k-\bar{x})(x_{k-\tau/\Delta t} - \bar{x})}{\sigma_{xx}^2} \qquad (5)$$

where $\bar{x}(t) = \frac{1}{N} \sum_{k=1}^{N} x_k(t)$

and

$$\sigma_{xx}^2 = \frac{1}{N} \sum_{k=1}^{N} (x_k - \bar{x})^2 .$$

The auto-correlation function and the normalized auto-correlation are related by

$$\phi_{xx}(\tau) = \sigma_{xx}^2 \; \phi'_{xx}(\tau) + (\bar{x})^2 \qquad (6)$$

The correlation functions are useful in describing a system's response in the time-domain.

Now, the frequency response of a system can be obtained by taking the Fourier transform of the correlation functions. Then useful reciprocal relations are known as Weiner's theorem. Namely, the power density spectrum, $\Phi(f)$, of a signal is the cosine Fourier transform of the auto-correlation function, $\phi(\tau)$.

This may be expressed as

$$\emptyset(\tau) = \int_{-\infty}^{\infty} \Phi(f) \exp(iw\tau)\, df, \quad w = 2\pi f$$

$$= 2 \int_{0}^{\infty} \Phi(f) \cos w\tau df \qquad (7)$$

where,

$$\Phi(f) = \lim_{T \to \infty} \frac{1}{T} |x(f)|^{2}$$

$$\Phi(f) = \lim_{\tau_m \to \infty} \int_{-\tau_m}^{\tau_m} \emptyset(\tau) \exp(-iw\tau)\, d\tau$$

$$= \lim_{\tau_m \to \infty} 2 \int_{0}^{\tau_m} \emptyset(\tau) \cos w\tau\, d\tau. \qquad (8)$$

It is evident that

$$\emptyset(0) = \overline{x^2} = \int_{-\infty}^{\infty} \Phi(f)\, df$$

is the total power, which by virtue of $\sigma^2 = \emptyset(o) - (\bar{x})^2$ is made up of an a-c. Component σ^2 and a d.c. component $(\bar{x})^2$.

The usefulness of the auto-correlation function therefore extends beyond its presentation of information in the time domain. By Fourier analysis the correlation function can give information in the frequency domain via the power spectrum.

5.3 Cross-Correlation Function and its Properties

When two random functions x and y are monitored, the cross-correlation is defined by the following equations:

$$\phi_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x(t)\ y(t+\tau)dt,$$

$$\phi_{yx}(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} y(t)\ x(t+\tau)dt.$$

This formula is a point-by-point multiplication of a waveform by the shifted second waveform, followed by an integration or summations over all time.

The following are properties of this function:

1. $\phi_{xy}(-\tau) = \phi_{yx}(\tau)$.

The cross-correlation displays symmetry about the ordinate when x and y are interchanged.

2. $|\phi_{xy}(\tau)|^2 \leq \phi_{xx}(0) \cdot \phi_{yy}(0)$ (i)

$$|\phi_{xy}(\tau)| \leq \frac{1}{2}[\phi_{xx}(0) + \phi_{yy}(0)] \quad \text{(ii)}$$

The equations define the bounding relationships for the magnitude of the cross-correlation function. The first states that the square of its magnitude is never greater than the product of the power contained in the two signals.

The second states that its magnitude is never greater than the average of the power contained in the two signals.

$$3. \quad \phi_{xy}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{xy}(f)\, e^{(i2\pi f\tau)}\, df \,.$$

The above equations states that a Fourier Transform pair relationship exists between the cross-correlation function and the cross power spectral density function. Unlike the Auto-correlation, the cross-correlation contains phase information besides amplitude. For digital techniques, the cross-correlation function is

$$\phi_{xy}(\tau) = \frac{1}{M} \sum_{k=1}^{M} x(t_k) \cdot y(t_{k+\tau})$$

where M is the number of points.

CHAPTER VI

ELECTRONIC INSTRUMENTS DESCRIPTION

6.1 Piezo-electric Transducer

Transducer is a device which converts one form of energy into another form energy.

For example loudspeaker is a transducer which converts electrical energy into acoustic energy. Microphone is a transducer which converts sound energy into electrical energy.

In this experiment a piezo-electric transducer which has a flat (± 3 dB) frequency response upto 2 MHz, made by ECIL is used to convert acoustic signal, caused by change in pressure when bubble collapses, into electrical signals. A Microphone which has a flat ± 3 dB frequency response upto 20 KHz, made by PHILIPS is also used to verify the signals that are obtained by ECIL transducer.

6.2 Charge Amplifier (Model 2735)

The instrument was designed and manufactured by ENDEVCO, Division of Becton, Dickinson and Company, U.S.A.

This charge amplifier is an all solid state instrument designed specifically to condition the signals from a transducer. It will amplify the signals which are generated by transducer.

6.3 Tunable Band-Pass Filter

Band-pass filter means it will pass only certain frequencies and reject other frequencies.

In this experiment KOMBINATIONS FILTER of type 01006 and type 01007 of VEBRFT MESSELEKTRONIK > OTTOSCHN < DRESDEN, West Germany is used as a high pass filter. It has tunable cut-off frequency arrangements. In the present experiment a frequency of 125 Hz is used for high pass filter.

6.4 Correlation and Probability Analyser (Model SAI-48)

The instrument was designed and manufactured by Signal Analysis Operation, Test Instruments Division, Honeywell, Denver, Colorado, U.S.A.

The SAI-48 provides on line real time computation of Auto and cross correlation functions with incremental lag or time delays ranging from 0.05 μs to 2 seconds resulting in maximum time delays from 20μs to 800 seconds. An Auto or cross-correlation function is determined sequentially at 400 incremental lag points so that a complete correlation function is displayed at one time.

In addition it has two other primary operating modes:

1. Probability (Density and Distribution)
2. Enhancement (or Signal Recovery).

In all modes the 400 analysis points are computed, and may be displayed on external equipment such as an

oscilloscope, or an X-Y recorder.

6.4.1 Correlation

Correlation analysis provides a quantitative measure of the degree of similarity between waveforms as they are being shifted relative to one another in time. If the signal is being compared with itself, the resulting waveform is an Auto-correlation. Any two different waveforms may be compared via cross-correlation.

The detection of signals in noise, the determination of dynamic system errors, automatic adjustment and control of processing plants, localization of interference sources, directional reception of signals, determination of speech signals and evaluation of ballisto cardiograms are a few possible applications. Model SAI-48 is used to find auto correlation in this experiment.

6.5 Fourier Transform Analyser (Model SAI-470)

The instrument was designed and manufactured by signal Analysis Operation, Test Instruments Division, Honeywell Inc., Denver, Colorado, U.S.A.

6.5.1. General Functional Description

The function of the Model SAI-470 is to compute the amplitudes of, and the frequencies present, in an input function

presented to it via an SAI-48.

The frequencies computed can be thought of as frequency vectors (i.e., complex frequencies) having a real and imaginary value, and therefore, a phase relationship.

Most specifically, if Φ_N represents the complex frequency magnitudes of some frequency f_n, then the SAI-470 solves the following basic equation:

$$\Phi_n(f_n) = \sum_{k=0}^{M} \phi_k(\tau) \left(e^{-i\frac{\pi kn}{N}}\right), \tag{1}$$

where k = Number of samples representing the input time function and k = 0,1,2,3,....M.

$\phi_k(\tau)$ = magnitude of the k-th input of the input time varying function, i.e. correlated function (which is in time domain)

$$e^{-\frac{i\pi kn}{N}} = \cos\frac{\pi kn}{N} - i \sin\frac{\pi kn}{N}$$

N = 1000 (maximum number of frequency intervals)

and n = 1,2,3,...1000.

Furthermore eqn.(1) can be expanded to include its real, A_n and its imaginary B_n, components where

$$A_n = \sum_{k=0}^{M} (\phi_k(\tau)) \cos\left(\frac{180kn}{N}\right) \tag{2}$$

$$\text{and } B_n = -i \sum_{k=0}^{M} (\phi_k(\tau)) \sin\left(180 \frac{kn}{N}\right) \tag{3}$$

From eqns.(2) and (3) the absolute frequency P_n magnitude can be computed:

$$P_n = \left[(A_n)^2 + (B_n)^2\right]^{1/2} \tag{4}$$

and the angle ϕ_n can be computed from

$$\tan \phi_n = \frac{B_n}{A_n} \tag{5}$$

or

$$P_n \sin \phi_n = B_n \tag{6}$$

To significantly smooth the power spectral density, the input function $\phi_k(\tau)$ is multiplied by a Hamming weighting function. This weighting function is defined as

$$WF = 0.54 \pm 0.46 \cos\left(\frac{360° k}{M}\right) \tag{7}$$

where M = total number of data samples (M = 400)

k = kth sample.

Also, $\phi_k(\tau)$ is multiplied by a scale factor of 1.28 to account for the fact that a maximum of 400 summations occur. The number 400 is closest to the binary number 512 (2^9) and therefore 512/400 = 1.28. This scale factor is actually taken into account in the weighting function as follows:

$$WF' = 1.28 \left[0.54 \pm 0.46 \cos\left(\frac{360^\circ k}{M}\right) \right] \qquad (8)$$

or

$$WF' = 0.6912 \pm 0.5888 \cos\left(\frac{360^\circ k}{M}\right)$$

where WF' is scale factored weighting function.

Therefore,

$$\phi_k'(\tau) = \phi_k(\tau) \left[0.6912 \pm 0.5888 \cos\left(\frac{360^\circ k}{M}\right) \right] \qquad (9)$$

$\phi_k'(\tau)$ is the smootnened auto-correlation function. The smootnened auto-correlation function is used in eqn. (1) to find power spectral density.

CHAPTER VII

EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experimental arrangement is explained in the following paragraphs.

A glass cylindrical chamber of size 4" diameter and 8" length is provided with an inlet and outlet. At the centre of the chamber a 2 m.m. hole is provided for air injection. In this chamber air is injected by means of a compressor at various air flow-rates. Now chamber is filled with water. When air is allowed to pass through the chamber air bubbles are produced. When air bubbles collapse, the acoustic signals are picked up by means of a piezo electric transducer and converted into electric signals. These signals are amplified by the charge amplifier.

The signals from the charge amplifier for various air flow-rates are displayed on oscilloscope. These signals are allowed to pass through a band-pass filter and the lower cut-off frequency was set at 125 Hz. Thus the 50 Hz a.c. components are eliminated. Now the signals that are coming from high pass filter due to bubble collapse are fed into signal correlator to compute auto correlation with delay time •5 m.s. Now the correlated function is in time domain. The correlated function is converted into

frequency domain by feeding the output of the correlator into a Fast Fourier Transform Analyser.

The output of the F.F.T., magnitude versus frequency is displayed on the oscilloscope. From the scope, the frequency of the acoustic signal due to bubble collapse is found.

The experiment is repeated for various water flow-rates (i.e. various pressure inside the chamber) and air-flow-rates. The pressure inside the chamber is measured by means of a U-tube manometer using mercury.

The schematic diagram of the experimental arrangement is shown in Figures (1) and (2).

FIGURE 1 : THE SCHEMATIC DIAGRAM OF ELECTRONIC INSTRUMENTS USED

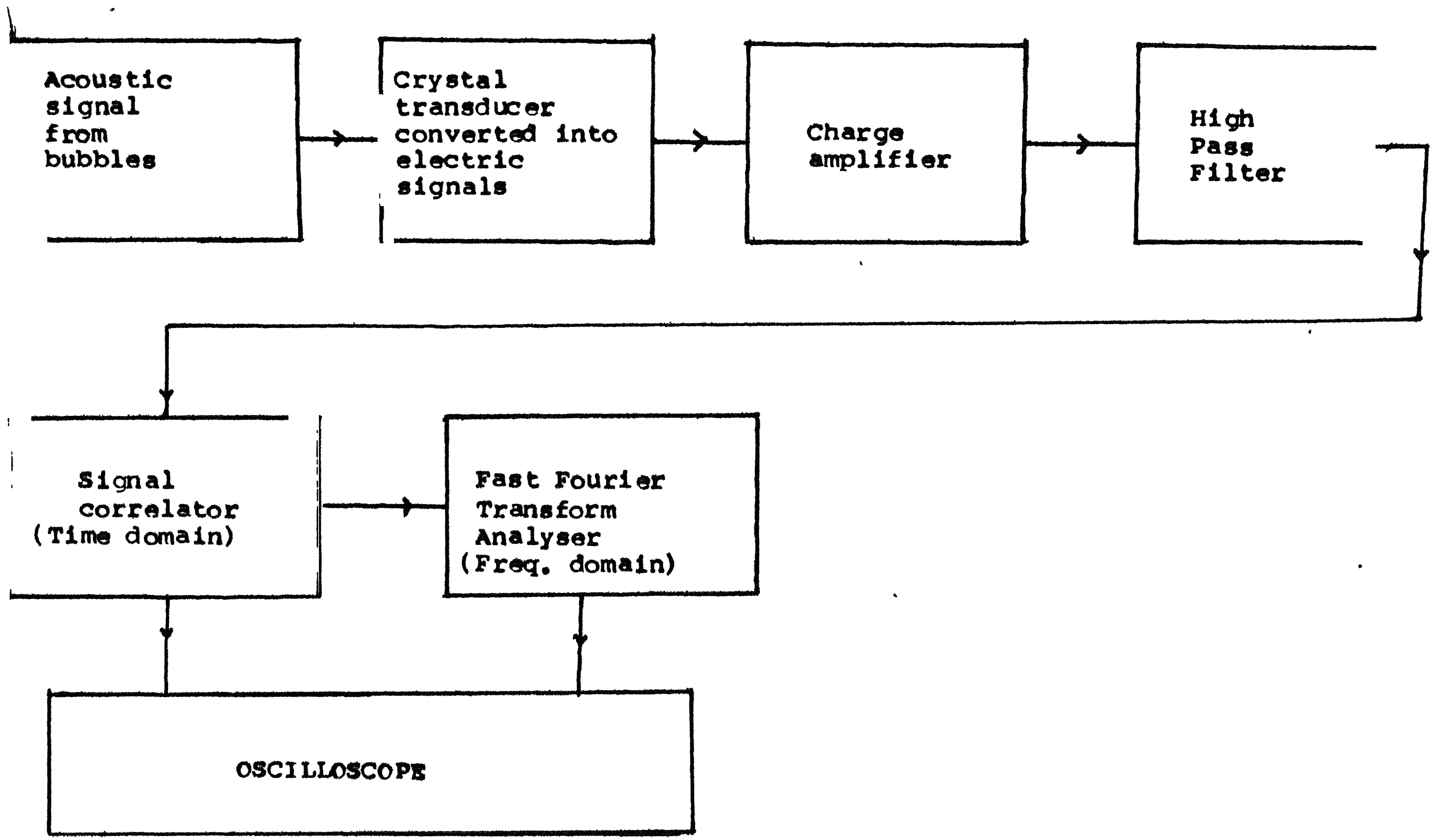
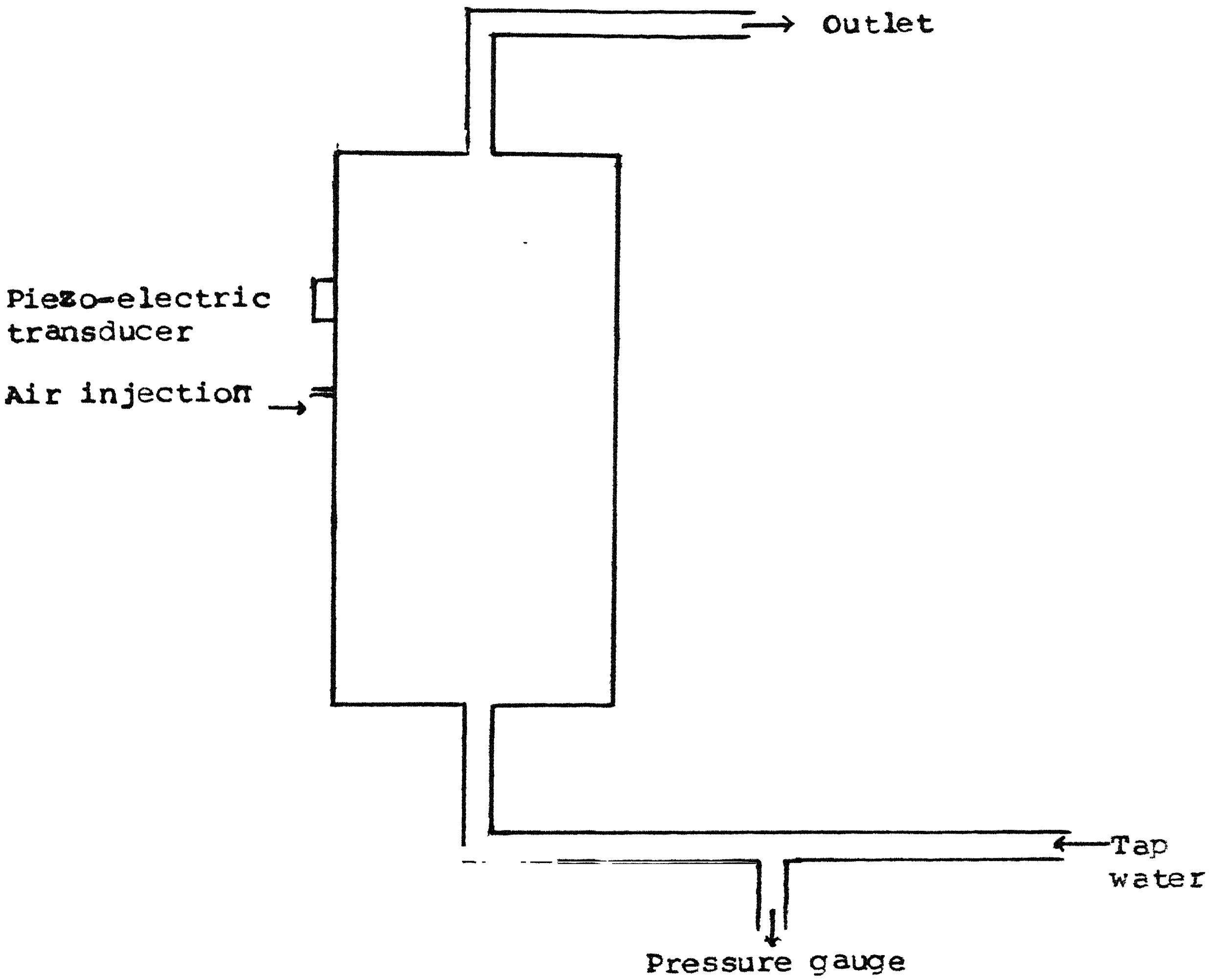


FIGURE 2 : ARRANGEMENT OF CHAMBER WITH TRANSDUCER



CHAPTER VIII

EXPERIMENTAL OBSERVATIONS AND TABULATIONS

Air is injected by means of a compressor into the chamber at various air-flow-rates in stationary water.

The power spectral density at various air-flow-rates in stationary water are tabulated in Table 1.

TABLE 1 : Frequency Spectrum of Air Injection into Stationary Water

Air Flow Rate cc/sec	With Bubbles Frequency (Hertz)	With Bubbles Mag. mV	Without Bubbles Frequency (Hertz)	Without Bubbles Magnitude mV
128.9 cc/sec	153	20	153	20
	203	25	203	25
	340	30		
	406	40		
	556	20		
257.8 cc/sec	153	20	153	30
	202	25	202	35
	320	50		
	410	30		
	553	40		
386.7 cc/sec	198	30	201	20
	335	20		
	408	35		
	543	40		
515.6 cc/sec	201	20	202	20
	331	30		
	410	35		
	524	40		
644.5 cc/sec	197	15	200	20
	325	30		
	528	25		

Then air is injected at various water flow-rates. The frequency spectrum (frequency versus magnitude) at each water flow rate for various air-flow rates are tabulated in Tables 2,3,4,5 and 6. The water flow rates are calculated from various pressures inside the chamber.

The calculations of water flow rates from various pressures are done by DEC-1090.Computer. The program and results are given. The problem of bubble formation and growth are discussed in Appendix-I. The calculations of radius of bubbles based on air-flow-rates are also discussed in Appendix-I.

TABLE 2 : Frequency Spectrum for 321.88 cc/sec. Water Flow Rate

Air Flow Rate	With Bubbles Frequency		Without Bubbles	
	Hertz	Magnitude	Freq. Hertz	Magnitude
128.9 cc/sec	148	50 mV		
	158	92.5 mV	158	100 mV
	187	75 mV		
	261 to 306	75 mV	200	80 mV
257.8 cc/sec	363	75 mV		
	392	92.5 mV		
	397	200 mV		
	552	50 mV	551	60 mV
	644 to 671	30 mV		
386.7 cc/sec	364 to 397	85 mV		
	552	50 mV	552	70 mV
	635 to 675	60 mV		
515.6 cc/sec	367 to 394	100 mV		
	650 to 670	40 mV		
	548	50 mV	552	50 mV

TABLE 3 : Frequency Spectrum for 352.61 cc/sec water flow-rate

Air Flow Rate	With Bubbles Frequency		Without Bubbles	
	Hertz	Magnitude	Freq. Hertz	Magnitude
128.9 cc/sec	391 to 400	30 mV	150	40 mV
	466 to 476	25 mV		
	646 to 756	60 mV		
257.8 cc/sec	360 to 408	65 mV	300	50 mV
	450 to 475	40 mV		
	648 to 750	60 mV		
386.7 cc/sec	348 to 454	70 mV	300	60 mV
	640	30 mV		
	680	30 mV		
	764	40 mV		
515.6 cc/sec	343 to 480	70 mV	305	55 mV
	640	45 mV		
	630	40 mV		
	765	50 mV		

TABLE 4 : Frequency Spectrum for 380.86 cc/sec. of water flow-rate

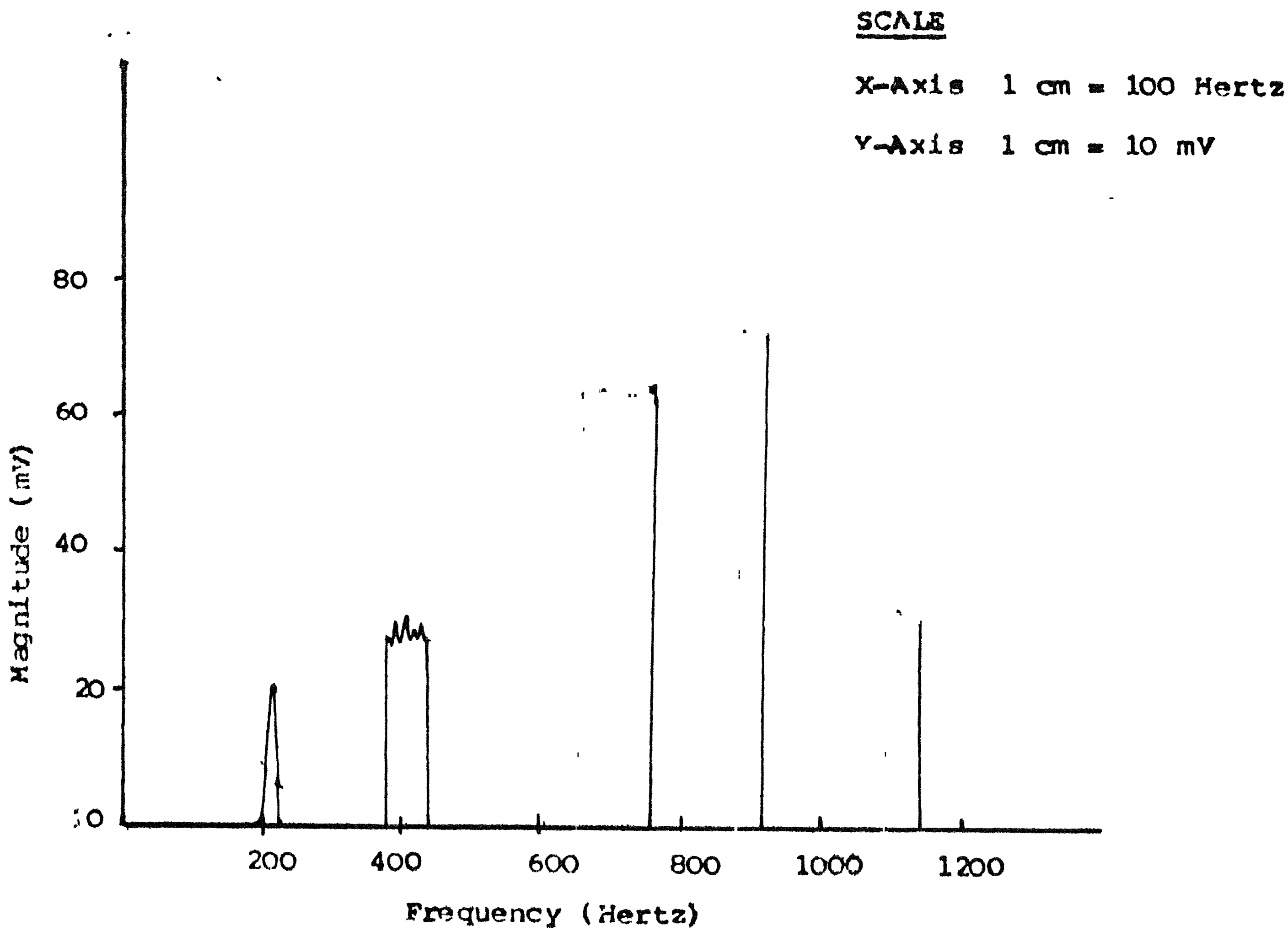
Air Flow Rate	With Bubbles Frequency		Without Bubbles	
	Hertz	Magnitude	Freq. Hertz	Magnitude
128.9 cc/sec	195 355 405 512 590 850	80 mV 40 mV 30 mV 80 mV 60 mV 30 mV	441	30 mV
257.8 cc/sec	150 197 402 470 575 852	30 mV 20 mV 45 mV 30 mV 30 mV 50 mV	436	35 mV
386.7 cc/sec	136 192 347 507 567 850 872	60 mV 40 mV 80 mV 35 mV 35 mV 40 mV 40 mV	438	35 mV
515.6 cc/sec	142 197 332 450 500 550 to 575 847 865	45 mV 30 mV 85 mV 30 mV 30 mV 35 mV 40 mV 40 mV	440	45 mV

TABLE 5 : Frequency Spectrum for 407.15 cc/sec of water flow-rate

Air Flow Rate	With Bubbles Frequency		Without Bubbles	
	Hertz	Magnitude	Freq. Hertz	Magnitude
	212	25 mV		
	385 to 437	25 mV	392	30 mV
128.9 cc/sec	437 to 450	20 mV		
	600 to 765	80 mV	552	80 mV
	875 to 900	80 mV		
	1097 to 1125	40 mV		
	216	20 mV		
	370 to 432	30 mV	393	30 mV
257.8 cc/sec	650 to 760	65 mV	551	70 mV
	870 to 907	75 mV		
	1097 to 1142	30 mV		
	210	30 mV		
	372 to 440	30 mV	394	30 mV
386.7 cc/sec	655 to 755	60 mV	553	65 mV
	870 to 908	75 mV		
	1097 to 1125	30 mV		
	210	25 mV		
	370 to 430	50 mV	390	30 mV
515.6 cc/sec	645 to 760	70 mV	552	70 mV
	870 to 910	70 mV		
	1085 to 1130	30 mV		

TABLE 6 : Frequency Spectrum for 431.85 cc/sec of water flow-rate

Air Flow Rate	With Bubbles Frequency		Without Bubbles	
	Hertz	Magnitude	Freq. Hertz	Magnitude
128.9 cc/sec	149	80 mV		
	255	40 mV		
	267	40 mV		
	400	30 mV		
	530	50 mV	565	30 mV
	642	40 mV		
	860	30 mV	880	30 mV
	1020	20 mV		
257.8 cc/sec	147.5	70 mV		
	240 to 260	30 mV		
	390 to 400	50 mV		
	550 to 570	40 mV	558	35 mV
	620 to 640	30 mV		
	850 to 860	40 mV	882	45 mV
	1020	30 mV		
386.7 cc/sec	152.5	60 mV		
	347.5	40 mV		
	380 to 400	50 mV		
	550 to 370	30 mV	555	50 mV
	620 to 640	30 mV		
	840 to 860	40 mV	887	35 mV
	1020	20 mV		
515.6 cc/sec	155	50 mV		
	350	50 mV		
	387.5	50 mV		
	437.5	40 mV		
	530.5	40 mV	550	50 mV
	562	40 mV		
	635	40 mV		
	840 to 860	50 mV		
	1014	30 mV	886	45 mV



A typical frequency spectrum of Bubbles for 257.8 cc/sec airflow rate in 407.15 cc/sec water flow-rate.

CHAPTER IX

RESULTS AND DISCUSSIONS

In the previous chapter, Table 1 gives frequency spectrum for various air flow-rates in stationary water. It is found that the frequencies did not vary much with various air-flow rates.

Strasberg [4] has shown that for air bubbles in water at atmosphere $fr = 330$ cm/sec (1)

where f - frequency, r - radius of bubble.

So for various radii of bubbles, the frequencies should vary as in eqn.(1).

But from Table 1 for various air-flow-rates, the frequencies did not vary. This indicates that there is no change in radius of bubbles.

For various water-flow-rates (from Tables 2,3,4,5 and 6) also, the frequencies did not vary with air-flow-rates.

Furthermore data of Tables 2,3,4,5 and 6 reveals that the frequencies are shifted with water flow-rates.

The shifted frequencies for each water flow-rates with and without bubbles are given in Table 7.

TABLE 7 : Frequency Shift with Water Flow Rates

With Bubbles

In stationary water. Frequency (Hertz)	With 321.88 cc/sec water flow-rate Frequency (Hertz)	With 352.61 cc/sec water flow-rate Frequency (Hertz)	With 380.86 cc/sec water flow-rate Frequency (Hertz)	With 407.15 cc/sec water flow-rate Frequency (Hertz)	With 431.85 cc/sec water flow-rate Frequency (Hertz)
-	-	-	-	200	380
-	-	-	197	360	530
138	148	-	330	402	630
363	350	460	570	700	850
408	540	630	-	880	1025
540	660	760	650	-	1130

Without Bubbles

With 321.8 cc/sec water flow-rate Frequency (Hertz)	With 352.6 cc/sec water flow-rate Frequency (Hertz)	With 380.86 cc/sec water-flow rate Frequency (Hertz)	With 407.15 cc/sec water flow-rate Frequency (Hertz)	With 431.85 cc/sec water flow-rate Frequency (Hertz)
200	295	441	550	-
380	500	610	760	875
-	150	-	390	550

From the water flow-rates, the velocity of water in the chamber can be calculated. The computer program with results are attached.

A graph of frequency versus 4th power of velocity of water is drawn (Fig. 3). It is found that in both cases they are linear.

It shows that the frequencies are shifting with 4th power of velocity of water.

That means the shifted frequencies are proportional to the 4th power of velocity of water.

So frequencies due to flow, i.e., without bubbles and frequencies due to bubbles shift to the 4th power of velocity of water

$$\text{i.e.} \quad f \propto v^4$$

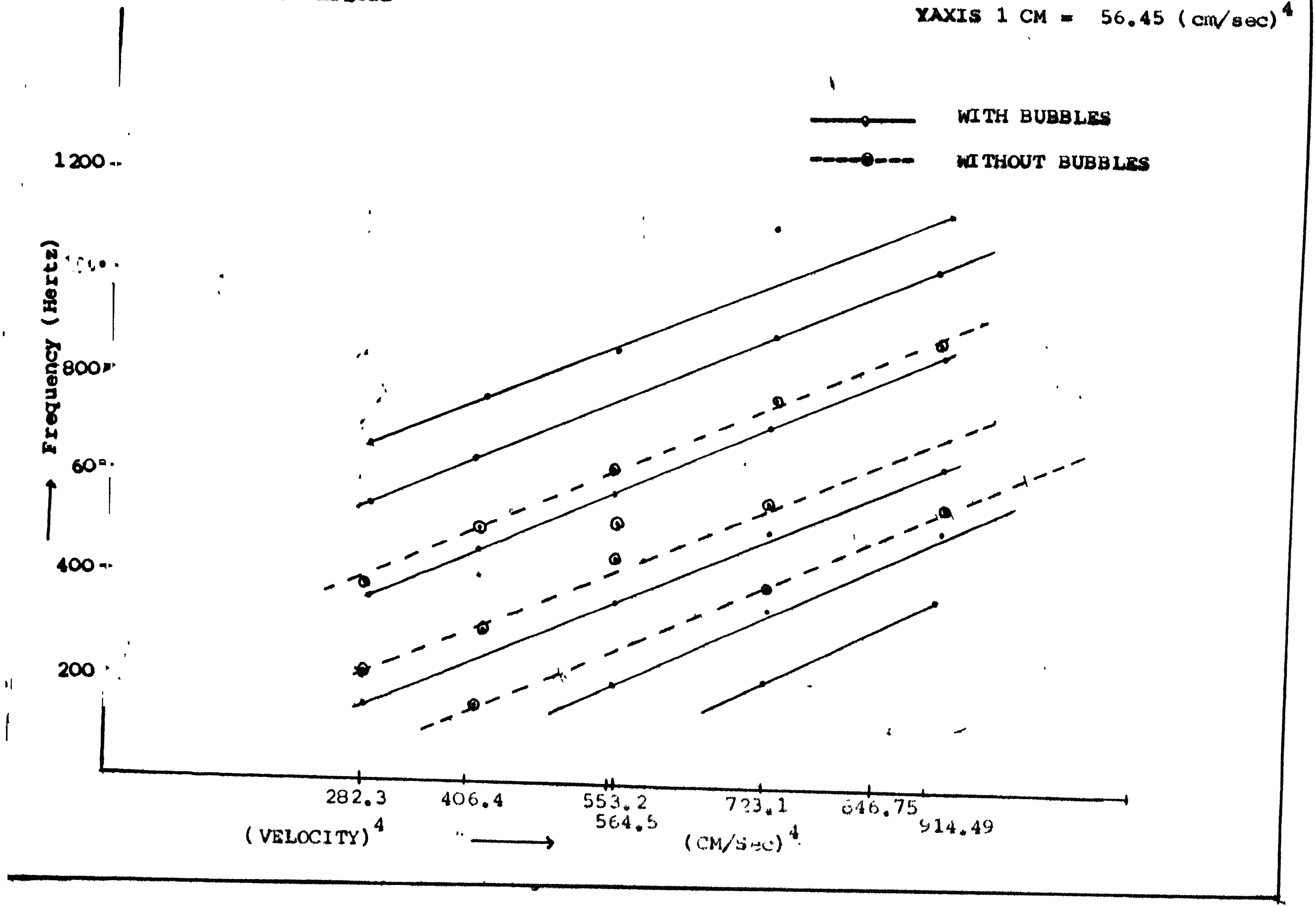
The curve fitting calculations between flow-rate and $(\text{velocity})^4$ for the datas in Table 7 are computed. The computer program with results are attached.

FIGURE 3 : PLOT OF FREQUENCY VERSUS (VELOCITY)^4 OF LIQUID

SCALE

XAXIS 1 CM = 100 Hz

YAXIS 1 CM = 56.45 (cm/sec)^4



CHAPTER X

CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions

From this experiment it is observed that

(1) The noise techniques can be used to obtain information about nucleate boiling. It is shown that the noise techniques can be used to determine the frequency response (spectral density) of bubbles in nucleate boiling for various flow-rates of coolant.

From the experimental observations it is found that

(1) The frequencies due to bubbles, frequency due to flow shifts to the 4th power of velocity of liquid.

(2) Some of the peaks at higher flow rates are due to frequency shift of the peaks present at low flow rates.

10.2 Recommendations

In this experiment frequencies due to bubble collapse are found. When the liquid is flowing all these frequencies are shifting to its 4th power of velocity of liquid. The dependence of shifting of frequency as a function of surface tension, viscosity and density is done by dimensional analysis which is discussed in Appendix II. The theoretical explanations are not yet studied. In future it can be studied.

REFERENCES

[1] Tigeat, Le Guillou, "Neutron Noise Induced by Vibration on the French Phenix LMFBR". Progress in Nuclear Energy. Vol. 1 (1977), page 487-496.

[2] Fry, D.N. and Robinson, J.C. CONF-670011.

[3] Guillou, Berger, Brunet, "Boiling Detection in Fast Reactors by Noise Analysis. Studies Performed in France". Progress in Nuclear Energy. Vol. 1 (1977), pages 409-427.

[4] M. Strasberg, 'Gas Bubbles as Sources of Sound in Liquids'. The Journal of the Acoustic Society of America. 28 (1956).

[5] Claytor, 'Measurement of Acoustic Noise Produced by Steam Injections into Sodium', Transactions of American Nuclear Society 43 (1982).

[6] T.N. Claytor, 'Measurement of the Sound Produced During Steam Injections into Sodium', Transactions of American Nuclear Society 30 (1978).

[7] S.P. Ying and C.C. Scott 'Significant Audible Noise from Sodium-Water Reactions'. The Journal of the Acoustic Society of America, 1970.

[8] Monideep Kumar DE, "Statistical Characteristics of Incipient two-phase Noise for Reactor Diagnosis".

[9] L.R. Gavrilov 'On the Size Distribution of Gas Bubbles in Water', Soviet Physics, 15, 1969.

[10] Joseph A. Thie, Reactor Noise. Rowman and Litterfield, Inc., New York (1962).

[11] Rice, Mathematical Analysis of Noise, Bell System Technical Journal, 23, 282 (1944).

[12] W.J. DUNCAN. Physical Similarity and Dimensional Analysis. Edward Arnold & Co. London (1953).

[13] E.W. Jupp. An Introduction to Dimensional Method. Cleaver-Hume Press Ltd., London (1962).

[14] M.M. ELWAKIL. Nuclear Power Engineering. McGraw-Hill Company, Inc., (1962).

APPENDIX I

Radius of Bubble Calculation

The problem of bubble formation and growth are discussed here to calculate the radius of bubble [14].

Figure (4.a) shows the bubble formation in a cavity. Let R to be the radius of cavity. Let 'r' to be the radius of bubble.

θ be the angle of contact between water and glass.

S be the surface tension of water.

P_f be the pressure of the liquid - dynes/cm^2

P_g be the pressure inside the bubble - dynes/cm^2

In Fig.(4.b) the bubble is free floating within a continuous liquid phase. The forces acting on that bubble are

(i) The force due to the pressure of the gas P_g (dynes/cm^2) inside the bubble $= \pi r^2 P_g$.

(ii) The force due to the pressure of the liquid P_f (dynes/cm^2) acting on the bubble $= \pi r^2 P_f$.

(iii) The surface tension force S (dynes/cm) on the liquid-gas interfaces of the bubble $= 2\pi RS \cos\theta$.

Considering one half of the bubble, a balance of the forces acting on it is represented by

$$\pi r^2 P_g = \pi r^2 P_f + 2\pi RS \cos\theta$$

$$r^2 = \frac{2\pi RS \cos\theta}{P_g - P_f} \qquad (1)$$

FIGURE 4(a) : BUBBLE FORMATION

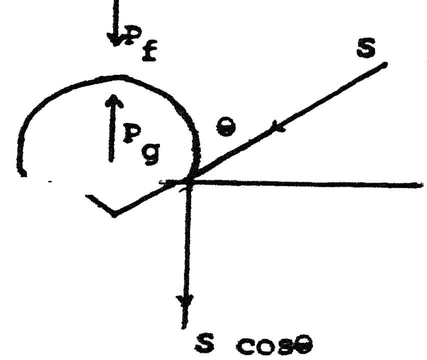
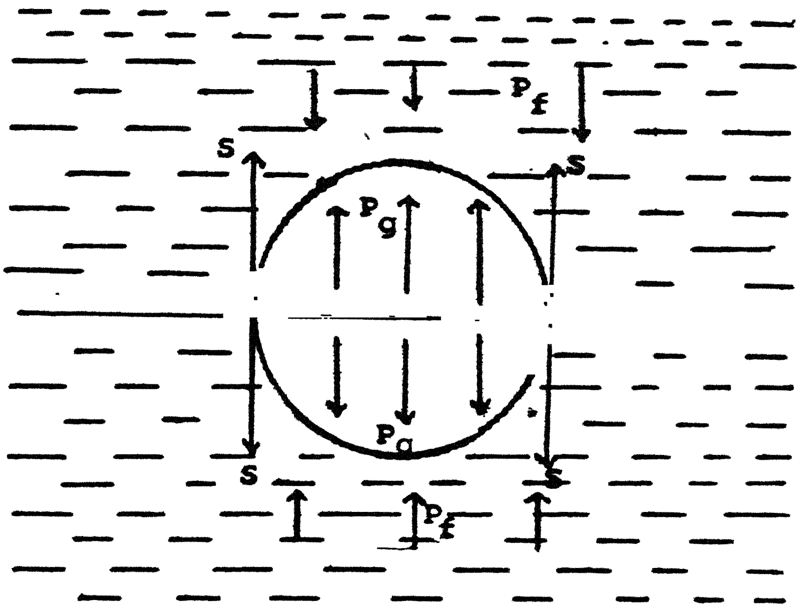


FIGURE 4(b) : FORCES ON A FREE BUBBLE WITH IN A LIQUID



In eqn.(1), R, S, θ are known. $(P_g - P_f)$ can be calculated from air-flow-rate. Therefore radius of the bubble can be calculated.

Now,

$$\text{Excess pressure inside the bubble i.e. } (P_g - P_f) = \left(\frac{\text{flow-rate}}{\text{area of cross-section of air flow}}\right)^2 \times \text{density of air} \qquad (2)$$

Substituting (2) in (1) radius of the bubble is calculated. The calculations of radius of bubbles based on air-flow-rates are done in DEC-1090 computer. The computer program with results are attached.

APPENDIX II

It is first thought the shifting of frequencies may be also as a function of surface tension, density, in addition with velocity of water. An attempt is made to explain this based on dimensional analysis (12) and (13).

Frequency $= f(\rho, S, v)$

$$\underline{f = k\,\rho^{a}\,S^{b}\,v^{c}} \qquad \text{------} \qquad (1)$$

v-velocity has dimension (LT^{-1})

ρ-density is mass per unit volume, (ML^{-3})

S-surface tension (i.e. Force Per Unit Length)

which has dimension $MLT^{-2}/L = MT^{-2}$

$$T^{-1} = (ML^{-3})^{a}\,(MT^{-2})^{b}\,(LT^{-1})^{c}$$

$$-1 = -2b - c \qquad (2)$$

$$0 = a + b \qquad (3)$$

$$0 = -3a + c \qquad (4)$$

Solving eqns.(2), (3) and (4) it is obtained that $a = 1$, $b = -1$, $c = +3$.

So eqn.(1) becomes $f = k\,\rho\,S^{-1}\,v^{3}$

$$f = \frac{k\,\rho\,v^{3}}{S}$$

By introducing one more parameter viscosity, in addition with surface, tension, density and velocity of the water, by dimensional analysis it obeys the formula $f = \dfrac{k\,\rho\,v^{4}\mu}{S^{2}}$ where μ viscosity of the liquid.

```
************************************************************************
*  PROGRAM TO CALCULATE THE FLOWRATE AND VELOCITY OF WATER FROM THE    *
*        MANOMETER HEIGHT                                              *
************************************************************************
*        TO FIND THE AREA OF CROSS-SECTION OF THE PIPE                 *
************************************************************************
         RADIUS = 0.63
         PI = 4.0 * ATAN(1.0)
         AREA = PI*RADIUS*RADIUS
************************************************************************
*        TO CALCULTE THE FLOW-RATE AND VELOCITY OF WATER               *
************************************************************************
         G = 980.0
         DHG = 13.6
         WRITE(35,20)
 20      FORMAT(//////////////,14X,'CALCULATION OF FLOW-RATE AND VELOCITY
         1 OF WATER',/)
         WRITE(35,30)
 30      FORMAT(10X,60('*'))
         WRITE(35,25)
 25      FORMAT(10X,'*      PRESSURE         *     FLOW-RATE    *      VEL
         1OCITY  *')
         WRITE (35,30)
         DO 10 I = 5,9
         HEIGHT = FLOAT (I)
         FRATE =SQRT(HEIGHT *AREA*AREA*G*DHG)
         VH20 = FRATE/78.54
         WRITE (35,50)HEIGHT,FRATE,VH20
 50      FORMAT(10X,'*',F8.2,2X,'CM OF Hg.',3X,'*',F8.2,' CC/SEC',
         13X,'*',F7.2,' CM/SEC  *')
 10      CONTINUE
         WRITE(35,30)
         STOP
         END
```

1234567890123456789012345678901234567890123456789012345678901234567890123456

CALCULATION OF FLOW-RATE AND VELOCITY OF WATER

PRESSURE	FLOW-RATE	VELOCITY
5.00 CM.OF Hg.	321.88 CC/SEC	4.10 CM/SEC
6.00 CM.OF Hg.	352.61 CC/SEC	4.49 CM/SEC
7.00 CM.OF Hg.	380.86 CC/SEC	4.85 CM/SEC
8.00 CM.OF Hg.	407.15 CC/SEC	5.18 CM/SEC
9.00 CM.OF Hg.	431.85 CC/SEC	5.50 CM/SEC

```
***************************************************************************
* PROGRAM TO FIND THE RADIUS OF BUBBLES FOR VARIOUS AIR FLOWRATES         *
***************************************************************************
        WRITE(22,25)
 25     FORMAT(2X, // // // // // // // /  )
        WRITE(22,10)
 10     FORMAT(9X,'CALCULATED RADIUS OF BUBBLES FOR AIR FLOWRATES',/)
        WRITE(22,30)
 30     FORMAT(8X,47('*'))
        WRITE(22,35)
 35     FORMAT(8X,'*  AIR FLOWRATE        *   RADIUS OF BUBBLES  *' )
        WRITE(22,30)
        DO 20 I = 1,5
        F = FLOAT(I)* 128.9
        R =0.25
        T = 75.0
        COSTHE = COSD(8.5)
        R2 = 2.0*R*T*COSTHE/(0.0254*F*F)
        RADIUS = SQRT(R2)
        WRITE(22,50)F,RADIUS
 50     FORMAT(8X,'*'F10.2,2X,'CC/SEC    *',F15.5,' CM.   *' )
 20     CONTINUE
        WRITE(22,30)
        STOP
        END
```

CALCULATED RADIUS OF BUBBLES FOR AIR FLOWRATES

AIR FLOWRATE	RADIUS OF BUBBLES
128.90 CC/SEC	0.29645 CM.
257.80 CC/SEC	0.14822 CM.
386.70 CC/SEC	0.09882 CM.
515.60 CC/SEC	0.07411 CM.
644.50 CC/SEC	0.05929 CM.

```
********************************************************************
*   THIS PROGRAM IS DESIGNED TO FIND THE LEAST SQUARES POLYNOMIAL    *
*   THAT ADEQUATELY REPRESENT A GIVEN SET OF DATA.THIS WILL FIT     *
*   LEAST SQUARES POLYNOMIAL UPTO AND INCLUDING DEGREE 10           *
********************************************************************
        DIMENSION A(11,12),XPX(11,11),XPY(11),X(20),Y(20),B(11)
        OPEN(UNIT=31,ACCESS = 'SEQIN',FILE='FOR23.DAT')
        OPEN(UNIT = 5,DEVICE= 'DSK',FILE = 'FOR33.DAT',ACCESS='SEQOUT')
        DO 121 INT = 1,9
1       READ(31,*)NUM
*******************************************************************
*  READ THE OBSERVATIONS AND CALCULATE THE COEFFICIENTS FOR THE     *
*  NORMAL EQUATIONS TO FIT A FIRST DEGREE POLYNOMIAL               *
*******************************************************************
        SUMY = 0.0
        SUMX = 0.0
        SUMXY = 0.0
        SUMXS = 0.0
        READ(31,*) ( X(I),Y(I),I=1,NUM)
        DO 4 I =1,NUM
        TYPE *, X(I),Y(I)
        SUMY = SUMY + Y(I)
        SUMX = SUMX + X(I)
        SUMXY = SUMXY + X(I)*Y(I)
        SUMXS = SUMXS + X(I)*X(I)
4       RMSSQ = 10.0 E08
        K =1
        N = K + 1
        M = N + 1
        XPX(1,1) = NUM
        XPX(1,2) = SUMX
        XPX(2,1) = SUMX
        XPX(2,2) = SUMXS
        XPY(1) = SUMY
        XPY(2) = SUMXY
******************************************************************
* TRANSFER THE XPX AND XPY QUANTITIES TO THE MATRIX AND SAVE THE   *
* XPX AND XPY QUANTITIES. THE GAUSS-JORDAN METHOD WILL PERFORM     *
* ELEMENTARY TRANSFORMATIONS UNTIL THE SOLUTION OF THE NORMAL      *
* EQUATIOINS IS IN A(I,M) , I=1,N.                                 *
******************************************************************
```

```
      18    DO 12 I = 1,N
            DO 10 J = 1,N
      10    A(I,J) = XPX(I,J)
      12    A(I,M)  = XPY(I)
*****************************************************************
*  SUBROUTINE CALLING TO SOLVE THE NORMAL EQUATIONS TO GET THE      *
*       LEAST SQUARES ESTIMATES OF THE PARAMETERS                   *
*****************************************************************
            CALL GSGOR(N,M,A)
            SUMSQ = 0.0
            DO 16 I = 1,NUM
            PROD = 0.0
            DO 17 J = 1,K
      17    PROD = PROD+A(J+1,M)*X(I)**J
            YHAT = A(1,M)+PROD
      16    SUMSQ = SUMSQ+(Y(I)-YHAT)**2
            IF(NUM-K-1.EQ.0) GO TO 42
*******************************************************************
*   CALCULATE THE CURRENT MODIFIED SUM OF SQUARES                   *
*******************************************************************
            CMSSQ = SUMSQ/(NUM-K-1)
*******************************************************************
* IF THE CURRENT MODIFIED SUM OF SQUARES IS GREATER THAN THE PREVIOUS *
* MODIFIED SUM OF SQUARES,AN ADEQUATE DEGREE POLYNOMIAL HAS BEEN    *
* OBTAINED, SO GO TO 42; OTHERWISE,WRITE THE DEGREE OF THE POLYNOMIAL *
* JUST OBTAINED, THE CORRESPONDING LEAST SQUARES ESTIMATES OF THE   *
* PARAMETERS,AND THE CURRENT MODIFIED SUM OF SQUARES                *
*******************************************************************
            IF(CMSSQ.GE.PMSSQ) GO TO 42
            WRITE (5,60)K
      60    FORMAT(/2X,'THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL
            1 OF DEGREE',I2,' ARE',/)
            DO 15 I = 1,N
            IM = I-1
      15    WRITE(5,65)IM,A(I,M)
      65    FORMAT(2X,'BETA(',I2,')=',F12.6)
            WRITE(5,70) CMSSQ
      70    FORMAT(/,2X,'CMSSQ=',F15.4,//)
            PMSSQ = CMSSQ
            IF(K.EQ.10) GO TO 1
```

```
*******************************************************************
* TNCREASE THE DEGREE OF POLYNOMIAL BEING FITTED,K,BUILD THE NEW      *
* POLYNOMIAL EOUATIONS,AND THEN RETURN TO STATEMENT 18                *
*******************************************************************
      K = K + 1
      N = K +1
      M = N + 1
      KP1 = K + 1
      KM1 = K - 1
      DO 20 I= 1,KM1
      XPX(KP1,I) = XPX(K,I+1)
20    XPX(I,KP1) = XPX(KP1,I)
      SUM1 = 0.0
      SUM2 = 0.0
      SUM3 = 0.0
      DO 21 I = 1,NUM
      XK = X(I)**K
      SUM1 = SUM1 + XK*X(I)**(K-1)
      SUM 2 = SUM2+XK*XK
21    SUM3 = SUM3 + XK*Y(I)
      XPX(KP1,K) = SUM1
      XPX(KP1,KP1) = SUM2
      XPX(K,KP1) = SUM1
      XPY(KP1) = SUM3
      DO 27J = 1,K
27    B(J)= A(J,N)
      GO TO 18
*********************************************************************
* PRINT  VELOCITY**4,FREQUENCY(EXPERIMENTAL),CALCULATED,ERROR FOR THE  *
* POLYNOMIAL ADEOUATELY REPRESENTING DATA                               *
*********************************************************************
42    KM1 = K-1
      WRITE(5,80)KM1
80    FORMAT(3X,//,3X,'THE POLYNOMTAL OF DEGREE',I3,1X,'ADEQUATELY
      1 REPRESENTS DATA')
      WRITE(5,85)
85    FORMAT(//4X,'(CC/SEC)**4'6X,'FREQUENCY',6X,'CALCULATED',3X,
      13X,'ERROR'/)
      DO 25 I = 1,NUM
      PROD = 0.0
      DO 26 J= 1,KM1
```

```
 26     PROD = PROD + B(J+1)*X(I)**J
        YHAT = B(1) + PROD
        DIFF = Y(I) - YHAT
 25     WRITE(5,90)X(I),Y(I),YHAT,DIFF
 90     FORMAT(4F15.6)
 121    CONTINUE
        STOP
        END
        SUBROUTINE GSGOR(N,M,A)
        DIMENSIONA(11,12)
        DO 16 K = 1,N
        KP1 = K+1
        DO 9 J = KP1,M
        A(K,J) = A(K,J)/A(K,K)
 9      CONTINUE
        DO 12 I = 1,N
        IF ( I .EQ.K) GO TO 12
        DO 14 J = KP1,M
        A(I,J) = A(I,J)- A(I,K)*A(K,J)
 14     CONTINUE
 12     CONTINUE
 16     CONTINUE
 13     RETURN
         END
```

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

```
BETA( 0)=  452.147750
BETA( 1)=    0.737672

CMSSQ=         89.6503
```

THE POLYNOMIAL OF DEGREE 1 ADEQUATELY REPPESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
282.540000	660.000000	660.569560	-0.569557
406.430000	760.000000	751.959720	8.040276
553.310000	850.000000	860.308960	-10.308960
915.060000	1130.000000	1127.161800	2.838242

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

```
BETA( 0)=  319.751980
BETA( 1)=    0.772799

CMSSQ=         18.4103
```

THE POLYNOMIAL OF DEGREE 1 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
282.540000	540.000000	538.098540	1.901459
406.430000	630.000000	633.840580	-3.840584
719.980000	880.000000	876.151640	3.848358
915.060000	1025.000000	1026.909200	-1.909225

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

```
BETA( 0)=  134.685480
BETA( 1)=    0.784262

CMSSQ=         30.0715
```

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 2 ARE

```
BETA( 0)=  113.239960
BETA( 1)=    0.867535
BETA( 2)=   -0.000069

CMSSQ=         24.1107
```

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 3 ARE

```
BETA( 0)=   52.615763
BETA( 1)=    1.232747
BETA( 2)=   -0.000732
BETA( 3)=    0.000000

CMSSQ=         19.2016
```

THE POLYNOMIAL OF DEGREE 3 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
282.540000	350.000000	350.790620	-0.790615
406.430000	460.000000	457.471110	2.528893
553.310000	570.000000	573.060750	-3.060753
719.980000	700.000000	698.354230	1.645767
915.060000	850.000000	850.323300	-0.323296

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

CMSSQ= 2000.6265

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 2 ARE

BETA(0)= 77.955549
BETA(1)= 0.123735
BETA(2)= 0.000512

CMSSQ= 1782.4162

THE POLYNOMIAL OF DEGREE 2 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
282.540000	148.000000	153.819940	-5.819944
553.310000	330.000000	303.291900	26.708099
719.980000	402.000000	432.656260	-30.656258
915.060000	630.000000	620.231900	9.768097

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

BETA(0)= -308.134980
BETA(1)= 0.919142

CMSSQ= 61.0085

THE POLYNOMIAL OF DEGREE 1 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
553.310000	197.000000	200.435610	-3.435612
719.980000	360.000000	353.629060	6.370945
915.060000	530.000000	532.935330	-2.935326

THE POLYNOMIAL OF DEGREE 0 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
719.980000	200.000000	353.629060	-153.629060
915.060000	380.000000	532.935330	-152.935330

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

BETA(0)= -29.609509
BETA(1)= 0.817648

CMSSQ= 237.4820

THE POLYNOMIAL OF DEGREE 1 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
282.540000	200.000000	201.408770	-1.408768
406.430000	295.000000	302.707180	-7.707180
553.310000	441.000000	422.803320	18.196678
719.980000	550.000000	559.080730	-9.080727

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

BETA(0)= 172.022330
BETA(1)= 0.787152

CMSSQ= 343.3217

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 2 ARE

BETA(0)= 81.285402
BETA(1)= 1.139484

THE POLYNOMIAL OF DEGREE 2 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
282.540000	380.000000	379.787900	0.212101
406.430000	500.000000	495.887630	4.112370
553.310000	610.000000	621.850260	-11.850258
719.980000	760.000000	749.434910	10.565086
915.060000	875.000000	878.039280	-3.039276

THE COEFFICIENT OF THE LEAST SQUARES POLYNIMIAL OF DEGREE 1 ARE

BETA(0)= -170.478360
BETA(1)= 0.781452

CMSSQ= 28.3857

THE POLYNOMIAL OF DEGREE 1 ADEQUATELY REPRESENTS DATA

(CC/SEC)**4	FREQUENCY	CALCULATED	ERROR
406.430000	150.000000	148.346440	1.653564
719.980000	390.000000	394.311340	-4.311337
915.060000	550.000000	547.342220	2.657776

A 92012

NETP-1986-M-KRI-DET